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PRODUCTION MAXIMIZATION: A SIMPLEX METHOD APPROACH (A CASE STUDY OF SAMFARERA RICE MILLS KANO-NIGERIA)

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ABSTRACT

This research paper optimized the rice production of Samfarera Rice Processing Mills using a famous mathematical technique known as Simplex method. The problems were transformed into linear programming and used the technique to determine the optimal solution. The company uses two machines in daily operations and produces some certain number of bags of process rice, the research provides an additional income by maximizing the product within stipulated time. The result validates hypothesis and found to be effective and efficient in maximizing the production leading to a significant business operations and high profit.

Keywords: Simplex Method; Elementary Row Operations; Dual Simplex Method; Production Maximization

INTRODUCTION

Samfarera is a small rice mill company established in the year 2015 with only two machines in operation daily. The company intends to minimize the number of days and cost of production to maximize its profits due to inflation that occurs occasionally, sometimes the company make a profit while at other times fail to do so, as a result of machines failure. The objective is to minimize the cost of production in setting up a suitable order for each machine with no harm, we use Simplex method to minimize the time and cost in order to maximize the production.

The Simplex method is a technique for solving linear programming models by a slack variable, tableaus, and pivotal variables for finding the optimal solution [1]. A linear programming is a procedure of acquiring the best outcome maximum or minimum value with linear constraints [2]. Most linear programming can be solved using an online solver such as MATLAB, Maple etc. However, the Simplex method is employed to solve linear programming by tableau. The method has a number of steps, in the first step the objective function Z should be generated and equivalent to the value at one vertex by graphical solution. In the next step the value of Z will be more accurate than previous one and also equal to the next adjoining vertex. Since, the number of vertices is finite; the technique also consists of finite number of steps for the optimal solution to achieved [3-6].

METHODOLOGY

To solve a linear programming problem using Simplex technique the following steps are necessary:

step 1: Write the equations in standard form

step 2: Present the slack variables

step 3: Construct the tableau

step 4: Locate the pivotal variable

step 5: Construct a new tableau

step 6: Checking for optimality

step 7: Determine optimal values

A minimization problem is said to be in standard form if the objective function

$$W = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \cdots + \beta_n x_n$$

will be equally minimize subject to the following constraints

$$\alpha_{11} x_1 + \alpha_{12} x_2 + \alpha_{13} x_3 + \cdots + \alpha_{1n} x_n \geq \lambda_1$$

$$\alpha_{21} x_1 + \alpha_{22} x_2 + \alpha_{23} x_3 + \cdots + \alpha_{2n} x_n \geq \lambda_2$$

…….…..

$$\alpha_{m1} x_1 + \alpha_{m2} x_2 + \alpha_{m3} x_3 + \cdots + \alpha_{mn} x_n \geq \lambda_m$$

where $x_i \geq 0$ and $\lambda_i \geq 0$

From the above inequalities, we form the argument matrix by adding a bottom row which consists:

the coefficients of the objective function

$$\begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots & \alpha_{1n} & | & \lambda_1 \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots & \alpha_{2n} & | & \lambda_2 \\ & & & \vdots & & & \\ \alpha_{m1} & \alpha_{m2} & \alpha_{m3} & \cdots & \alpha_{mn} & | & \lambda_m \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \beta_1 & \beta_2 & \beta_3 & \vdots & \beta_n & | & 0 \end{bmatrix}$$

The transpose of the above matrix is

$$\begin{bmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} & \cdots & \alpha_{m1} & | & \beta_1 \\ \alpha_{12} & \alpha_{22} & \alpha_{32} & \cdots & \alpha_{m2} & | & \beta_2 \\ & & & \vdots & & & \\ \alpha_{1n} & \alpha_{2n} & \alpha_{3n} & \cdots & \alpha_{mn} & | & \beta_n \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \lambda_1 & \lambda_2 & \lambda_3 & \vdots & \lambda_m & | & 0 \end{bmatrix}$$

Therefore, the Dual maximization problem corresponding to this transpose matrix is given by

$$Z = \gamma_1 y_1 + \gamma_2 y_2 + \gamma_3 y_3 + \gamma_4 y_4 + \cdots + \gamma_n y_n$$

Subject to the following constraints

$$\alpha_{11} y_1 + \alpha_{12} y_2 + \alpha_{13} y_3 + \cdots + \alpha_{1n} y_n \geq \lambda 1$$

$$\alpha_{21} y_1 + \alpha_{22} y_2 + \alpha_{23} y_3 + \cdots + \alpha_{2n} y_n \geq \lambda 2$$

$$\vdots$$

$$\alpha_{m1} y_1 + \alpha_{m2} y_2 + \alpha_{m3} y_3 + \cdots + \alpha_{mn} y_n \geq \lambda m$$

$$\text{where } y_i \geq 0 \text{ and } \lambda_i \geq 0$$

Then, by applying the Simplex technique, the Dual maximization problem gives the maximum value of Z which turn to be a minimum value of W. Some of the minimization methods are as follows:

Northwest corner method

Vogel approximation method

Least cost method

Simplex Method

Simplex method is a technique for solving Linear programming problem models with two or more decision variables [7-11]. The variables are:

Basic variables: Are the variables with non-zero coefficient in the constraints of the linear programming problem [12].

Non-Basic variables: Are the variables with coefficients taking any of the values, whether positive, negative or zero.

Slack, surplus and artificial variables:

If the inequality is $\leq$ then, add a slack variable i.e. +S to change the sign from $\leq$ to $=$

If the inequality is $\geq$ then, subtract a surplus variable i.e. S to change the sign from to $=$

If they are $=$ use artificial variables

Algorithm: Simplex Method

Determine a starting basic feasible solution.

Select an entering variable using the optimality condition. Stop if there is no entering variable.

Select a leaving variable using the feasibility condition [7].

Optimality Condition

The entering variable in a maximization or minimization problems is the non-basic variable having the most negative coefficient in the Z row. The optimum is achieved at the iteration where all the Z-row coefficients of the non-basic variables are non-negative [7,12-15].

Feasibility Condition

For both maximization and minimization problems the leaving variable is the basic associated with the smallest non-negative ratio [16-18].

Pivotal Row

The procedures to determine the pivotal row are as follows:

Replace the leaving variable in the basic column with the entering variable.

New pivot row is equal to the current pivot row divided by the pivot element.

All other rows: new row $=$ current row $-$ (pivot column coefficient) $\times$ new pivot row.

Samfarera Rice Mills Company has two machines for daily operations. Machine 1 costs $2,000 which is equivalent to ₦2,000,000 per day to operate, and it can produce 40 bags of grade (A) rice, 30 bags of grade (B) rice, and 20 bags of grade (C) rice. Machine 2 is a new model which costs $2,500 equivalents to ₦2,500,000 per day to operate and it can produce 30 bags of grade (A) rice, 40 bags of grade (B) rice and 50 bags of grade (C) rice. The company has order from the customers' amount to 2,500 bags of grade (A) rice, 2,700 bags of grade (B) rice and 3,000 bags of grade (C) rice. The problem is to determine how many days should each machine run to minimize the cost maximize the production and mills enough rice to meet the demand of its customers.

Let x1 and x2 be the number of days that the two machines are operated. The total cost is given by the objective function Z = 2, 000x1 + 2, 500x2 and the constraints are:

here x1 $\geq$ 0 and x2 $\geq$ 0

$$40x_1 + 30x_2 \geq 2,500$$
$$30x_1 + 40x_2 \geq 2,700$$
$$20x_1 + 50x_2 \geq 3,000$$

The argument matrix correspond to the minimization problem is as follows

$$\begin{bmatrix} 40 & 30 & | & 2,500 \\ 30 & 40 & | & 2,700 \\ 20 & 50 & | & 3,000 \\ \cdots & \cdots & | & \cdots \\ 2,000 & 2,500 & | & 0 \end{bmatrix}$$

The transpose matrix correspond to the minimization problem is given as

$$\begin{bmatrix} 40 & 30 & 20 & | & 2,000 \\ 30 & 40 & 50 & | & 2,500 \\ \cdots & \cdots & \cdots & | & \cdots \\ 2,500 & 2,700 & 3,000 & | & 0 \end{bmatrix}$$

Apply Simplex method over the Dual simplex problem

Table 1: Simplex Tableau 1

Basic variable	y_1	y_2	y_3	S_1	S_2	β
S_1	40	30	20	1	0	2,000
S_2	30	40	50	0	1	2,500
W	$-2,500$	$-2,700$	-3,000	0	0	0

$$\frac{2,000}{20} = 100 \text{ and } \frac{2,500}{50} = 50$$

Now, the variable 50 is our pivot variable, then apply the Simplex method over the Dual problemagain, we have:

Table 2: Simplex Tableau 2

Basic variable	y_1	y_2	y_3	S_1	S_2	β
S_1	28	14	0	1	$-\frac{2}{5}$	1,000
y_3	$\frac{3}{5}$	$\frac{4}{5}$	1	0	$\frac{1}{50}$	50
W	-700	-300	0	0	60	150,000

$$\frac{1,000}{28} = 35.7 \text{ and } \frac{50}{\frac{3}{5}} = 83.3$$

Repeat the above procedures, now 35.7 variable is our new pivot variable gives:

Table 3: Simplex Tableau 3

Basic variable	y_1	y_2	y_3	S_1	S_2	β	β
y_1	1	$\frac{1}{2}$	0	$\frac{1}{28}$	$-\frac{1}{70}$	$\frac{250}{7}$	,000
y_3	0	$\frac{1}{2}$	1	$-\frac{3}{140}$	$\frac{1}{35}$	$\frac{200}{7}$	50
W	0	50	0	25	50	175,000	0,000

$$\frac{1,000}{28} = 35.7 \text{ and } \frac{50}{\frac{3}{5}} = 83.3$$

Therefore, a minimization problem implies that the cost of rice production from the two machines are amount to \$175,000 which is equivalent to ₦175,000,000 and this occurs when $x_1 = S_1 = 25$ and $x_2 = S_2 = 50$. However, the two machines operated for the following number of days: -

- Machine 1 = 25 days
- Machine 2 = 50 days

Experimental Results

In table 1 and 2, it is observed that the tables are not optimal since the last row contains a negative value, so that the procedures should be repeated until it becomes 0. Table 3, shows that the optimal solution and the cost of production can be identified at minimal cost. The two machines produced the following number of bags of rice

(1) Grade(A) $\Rightarrow (40 \times 25) + (30 \times 50) = 2,500$ bags of rice

(2) Grade (B) $\Rightarrow (30 \times 25) + (40 \times 50) = 2,750$ bags of rice

(3) Grade (C) $\Rightarrow (20 \times 25) + (50 \times 50) = 3,000$ bags of rice

Therefore, the required order made by the customers has been achieved with the surplus of 50 bags of grade (B) rice.

CONCLUSION

Based on the findings of this study, the minimization of cost and maximization of rice production by Simplex method and Dual method met the demand of the company customers. The technique improves the company production with additional 50 bags of grade (B) Rice. The Method was found to be worthy and efficient in maximizing the product and equally guarantees the profitability of the company.

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